Nonparametric, Model-Based Radial Lens Distortion Correction Using Tilted Camera Assumption *

Janez Perš, Stanislav Kovačič

Faculty of Electrical Engineering

University of Ljubljana

Tržaška 25, SI-1000 Slovenia

Tel: +386 1 4768 876   Fax: +386 1 4768 279

e-mail: {janez.pers},{stanislav.kovacic}@fe.uni-lj.si

Abstract

Radial lens distortion prohibits use of simple pinhole camera models in computer vision applications, especially when using wide-angle lenses, which result in barrel-type distortion. Usual approach to radial distortion is by the means of polynomial approximation, which introduces distortion-specific parameters into the camera model and requires iterative methods for their calculation. Based on the properties of distorted images, an alternative approach is proposed in this paper. The basic assumption is that distortion occurs due to transformation of the observed differential of radius and is locally dependent of the angle of principal rays. The geometric relations which result from this assumption are complemented with the equations of the perspective radial lens projection function to derive model of radial distortion with single parameter - focal length. Experiments were conducted to illustrate the validity and performance of this approach.

Key words: lens distortions, radial distortion, camera calibration

1 Introduction

Lens *distortions* are long-known phenomena that prohibit use of simple pinhole camera models in the most of the computer vision applications. Being the most stubborn type of lens *aberrations*, they do not influence quality of the image, but have significant impact on image geometry [4]. Several types of lens distortions exist, however, *radial* distortion is usually the most severe part of the total lens distortion, especially when inexpensive

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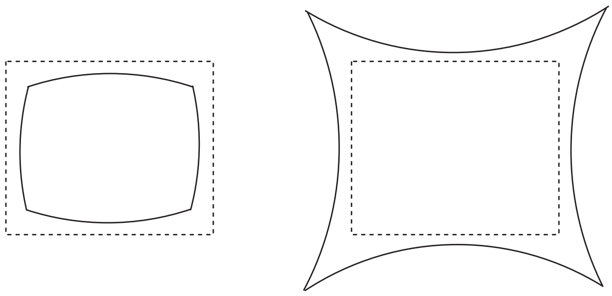
wide-angle lenses are used. Effect of radial distortion on image geometry is illustrated in Figure 1.



(a) (b)

Figure 1: Effect of radial distortion on image geometry. Dashed line represents the rectangular object as it would appear in absence of radial distortion. Solid line shows the object shape in the presence of (a) barrel and (b) pinchusion distortion.

There are two major types of radial distortion [4]. When image points get displaced from its desired location to the position closer to the optical axis (negative displacement), *barrel* distortion occurs. Alternatively, image points can get displaced to the position further away from the optical axis (positive displacement), in this case *pinchusion* distortion occurs. Barrel distortion is common in wide angle lenses and it therefore dominates the distortion-related research, as far as computer vision is considered.

1.1 Related work

The science of precise measurement using optical instruments has developed long time before first computer vision-based measuring systems became available [4]. Major part of photogrammetric work was performed manually and high-quality optical equipment was prerequisite for accurate measurements. These instruments are today referred to as "metric" equipment, in contrast to "non-metric" or consumer equipment, which now dominates the field of computer vision. Expensive metric cameras usually incorporated complex optics, which included correcting elements aimed at correcting lens distortions. Radial distortions of these cameras were small (in the range of micrometers), but were nevertheless fully documented in camera documentation [4]. Calibration of these high-precision cameras was performed using highly specialized equipment.

Advent of computer vision brought off-the-shelf cameras and lenses into the field of visual inspection and measurement. This required different calibration procedures, which could be carried out on inexpensive, but computer-supported equipment. Radial distortions of these lenses were much higher (several percent at the image boundary, see [8]). Polynomial model for radial distortion, which originated in photogrammetry was adopted, as demonstrated by Tsai [8].

In the following years, many authors tried to compensate for radial lens distortion. Some of them used wide-angle lens for image acquisition, which resulted in radial distortions evidently exceeding 20% [7]. This called for some kind of radial distortion correction even when no precise measurements were performed. Most approaches used polynomial approximation model for radial distortion, with rare exceptions [1], such as FET (Fish-Eye Transform) model by Basu and Licardie and FOV (Field-Of-View) model by Devernay and Faugeras. The FET model is based on the observation that fish-eye have a high resolution at the fovea, and a non-linearly decreasing resolution towards the periphery. FOV model is based on simple optical model of fish-eye lens and introduces field of view ω as distortion parameter. However, neither FET nor FOV model provides relations between the distortion parameters and the *physically measurable* lens parameters.

Most of the radial distortion-focused research is still based on polynomial models and their variations, for example [2].

1.2 Our approach

Several authors label radial distortion as an *error* of the lens design and manufacturing. However, it is inherent property of any lens [3] and has to be compensated for, either mathematically or optically. In this paper we derive a mathematical model of radial distortion which is based on the camera and lens projection geometry and does not introduce *any* distortion-specific parameters into the camera model.

This paper is structured as follows: first, we define ideal (linear) camera model and expand it with the polynomial-based (classical) radial distortion model. Next, we present a concept of radial projection function, which is used in lens design and mathematics to study lens properties. In the next step, we propose new approach for modeling lens distortion, which is subsequently used to derive alternative model of radial distortion, based on the most widely used, *perspective* projection function. Next, we present some results of tests on the real images, that demonstrate the effectiveness of this approach, and finally we conclude the paper with comments on properties of this alternative radial distortion model.

2 Linear camera and polynomial distortion model

Pinhole camera can be represented by the following linear model [3]:

$$\begin{bmatrix} \mathbf{u}_l \\ 1 \end{bmatrix} \simeq \mathbf{K}[\mathbf{R} \mid -\mathbf{R}\mathbf{t}] \begin{bmatrix} \mathbf{X} \\ 1 \end{bmatrix}, \tag{1}$$

where $\mathbf{u}_l = [u_l, v_l]^\top$ are the coordinates of the image point, which is a projection of a scene point with coordinates $\mathbf{X} = [X, Y, Z]^\top$. $\mathbf{K}$ is the calibration matrix, containing all intrinsic camera parameters. $\mathbf{R}$ and $\mathbf{t}$ are rotation matrix and translation vector, the extrinsic camera parameters. This is an idealized camera model - $\mathbf{u}_l$ cannot be directly measured due to distortions.

Therefore, we can extend the linear camera model with radial distortion, which could be generally represented as [3]:

$$\mathbf{u} = d(\mathbf{u}_l, \mathbf{p}), \tag{2}$$

where $\mathbf{u}_l$ are the coordinates of the undistorted image point and $\mathbf{u} = [u, v]^\top$ are the coordinates of the distorted image point. d is the distortion function and $\mathbf{p}$ is the vector of distortion parameters. $\mathbf{u}$ is directly measurable, for example as distances on the surface of the photographic film, or as coordinates of a pixel in an image from the CCD camera.

Polynomial approximation has been thus far the preferred method of modeling radial distortion function d. Polynomial model of radial distortion can be then expressed by the following equations:

$$\frac{r}{r_l} = \frac{\|\mathbf{u} - \mathbf{u_0}\|}{\|\mathbf{u_l} - \mathbf{u_0}\|} = \frac{u - u_0}{u_l - u_0} = \frac{v - v_0}{v_l - v_0} = D(r_l, \mathbf{k}), \tag{3}$$

where the polynomial on the right-hand side is given by

$$D(r_l, \mathbf{k}) = 1 + k_1 r_l^2 + k_2 r_l^4 + \ldots + k_n r_l^{2n}. \tag{4}$$

The point $\mathbf{u}_0 = [u_0, v_0]^\top$ is the image center, $\mathbf{k} = [k_1, k_2 \ldots k_n]^\top$ are the distortion coefficients and $r_l = \|\mathbf{u}_l - \mathbf{u}_0\| = [(u_l - u_0)^2 + (v_l - v_0)^2]^{(1/2)}$ is the radius of $\mathbf{u}_l$, or the distance from $\mathbf{u}_l$ to $\mathbf{u}_0$. The length of the vector $\mathbf{k}$ is denoted by n, and $2n$ is the order of the distortion polynomial D. Most of the authors, including [8], concluded that for most of the practical tasks, second order ($n = 1$) or fourth order ($n = 2$) polynomial is sufficient. However, this may not be case for wide-angle lenses, as illustrated in [6]. In addition, independently of the number of the coefficients used, this model of radial distortion *always* requires iterative approach to obtain the coefficients, which is its major drawback. Furthermore, if the inverse of function $d(\mathbf{u}_l, \mathbf{p})$ is needed it has to be computed iteratively as well [3].

3 Camera model-based radial distortion function

Polynomial approximation of radial distortion function, as defined in Equations (3) and (4) is based on the assumption that the underlying distortion function is not known and that it cannot be obtained by analytical means. This is probably true for high-quality lens with distortion-correcting elements, where polynomial function approximates the inaccuracies in the lens manufacturing. However, this may not be true for simple, widely-used lenses which have significant distortions that result from the lens geometry itself.

3.1 Camera models

The projection geometry of most cameras can be modeled as perspective projection of the 3D world onto a sphere (the *viewing sphere*), followed by a projection of the sphere onto a plane [5]). Five ideal *radial projection functions* are used in lens design and mathematics to map an angular distance α from the optical axis onto a distance $r(\alpha)$ from the image center $\mathbf{u}_0$, [5]: perspective, $r(\alpha) = k \tan \alpha$, stereographic, $r(\alpha) = k \tan(\alpha/2)$, equidistant,

$r(\alpha) = k\alpha$, equi-solid angle, $r(\alpha) = k \sin(\alpha/2)$ and sine law, $r(\alpha) = k \sin(\alpha/2)$. The coefficient k corresponds to the focal length f of the lens used for image acquisition, [4].

3.2 Correcting the distortion

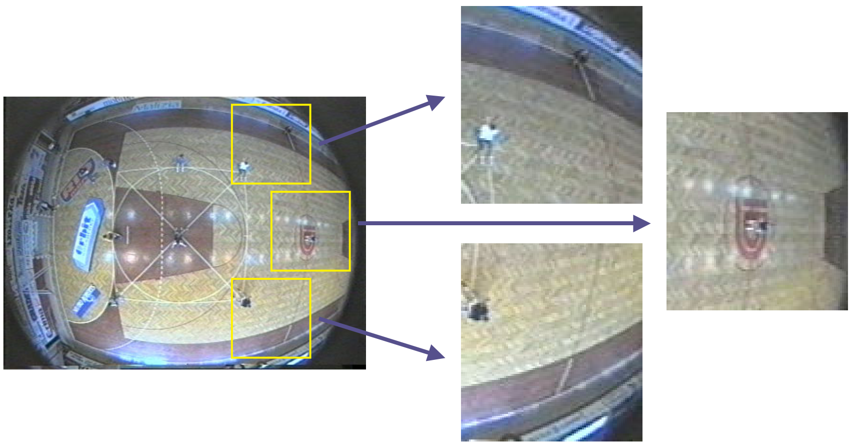
We can formulate our problem as follows: we are looking for the radial distortion function d, as defined in Equation (2). To stress the radially symmetric nature of d, we can rewrite it as

$$r' = d(r'_l, \mathbf{p}), \tag{5}$$

where $r'_l = \|\mathbf{u'}_l - \mathbf{u'}_0\| = [(u'_l - u'_0)^2 + (v'_l - v'_0)^2]^{(1/2)}$. The prime signs denote the variables which are defined on the image plane, not in the object space.

Although polynomial approximation is sometimes thought of as being the only way to model the unknown distortion function, this is not the case. Model of ideal camera can be changed to incorporate the radial distortion, even if such model does not correspond closely to the actual physics of the real camera. Example of such approach is the principle of *variable focal length* [4], which assumes that focal length of the camera changes with respect to the the radial distance r_l, which causes radial distortion. This principle was successfully employed in certain types of photogrammetric instruments [4].

Similarly, we propose another model of radial lens distortion, which can be constructed by observing the effects of radial distortion on images, acquired using wide-angle lens. Let us look at the typical, barrel-distorted image, shown in Figure 2a. Significant distortion manifests itself through intense bending of otherwise straight lines of the planar pattern.



(a) (b)

Figure 2: (a) Image of the planar pattern (handball/basketball court), acquired using wide-angle lens. (b) Three enlarged sections of the original image look similar as they were acquired with the tilted camera, using lens with the smaller viewing angle.

Closer look at the three enlarged sections of the original image, shown in Figure 2b reveals similar appearance, as if these images were acquired separately, with the help of tilted camera, using lens with the smaller viewing angle. If tilted camera would be used, distances on these images would appear shorter than they are on the observed plane due to tilt. Then, the following assumption can be formulated:

Assumption. *Barrel distortion of wide-angle lens occurs due to the transformation of radius on the observed plane to the radius on the image plane under the influence of the viewing angle* α.

Certain geometric relations can be established on the basis of this assumption, as illustrated in Figure 3.

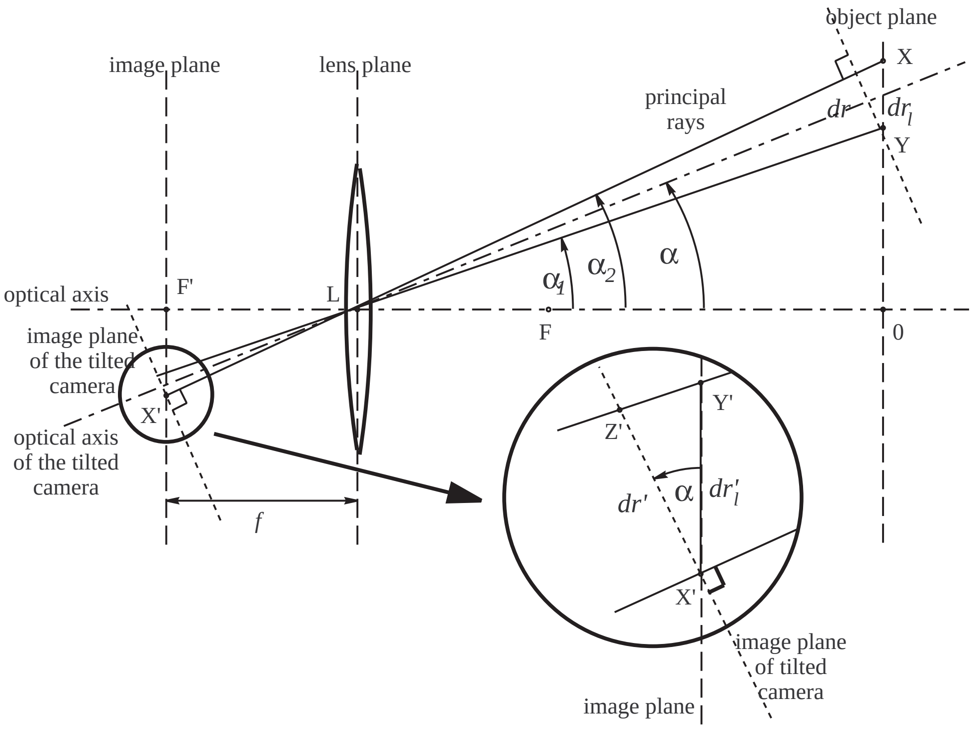


Figure 3: Geometry related to tilted camera assumption.

Position of the observed point on the image plane is marked by X, and its image on the image plane is marked X'. We can then define the following quantities: $\overline{F'L} = \overline{FL} = f$, the focal length of the lens. $\overline{OX} = r_l$ is the true radial distance of the point X in object plane. $\overline{XY} = dr_l$ is the length of differential of the radius in the object plane. Similarly, $\overline{X'Y'} = dr'_l$, is the length of the image of the radius differential dr_l in the image plane. α_1 and α_2 are angles of principal rays originating at the opposite ends of dr_l and ending at the opposite ends of dr'_l. Let us additionally assume the position of observed object in infinity[1], which causes the image plane to appear exactly at the focal point F'. We also assume that the observed dr_l part of radius r_l in the object plane is infinitely small. As a consequence, principal rays are parallel, therefore $\alpha_1 = \alpha_2 = \alpha$.

[1]In practice, this means that object distance is much larger than focal length f of the lens used, $\overline{LO} \gg f$.

To account for radial distortion with accordance to the tilted camera assumption, we introduce the concept of imaginary *tilted camera*, which is tilted for angle α from optical axis of our (real) camera, observing the point X on the image plane. Optical axis of tilted camera is intersecting with the object plane near the point X - it is intersecting the plane *exactly* at the point X if dr_l is infinitely small.[2] Similarly, let us also assume that the image plane of tilted camera intersects with the image plane of real camera exactly in point X'. The radial distance r of point X' on the image to the image plane center F' is equal to $\overline{F'X'}$. Due to camera tilt α, the differential dr'_l is then projected to the image plane of our tilted camera according to the following formula:

$$dr' = \cos(\alpha) \cdot dr'_l, \tag{6}$$

as shown in the enlarged part of the Figure 3. By changing the angle α, we can obtain distortion for every differential $dr'(r')$, along the radius r'. Relation between the angle α and radial distance r' along the image plane can be obtained from camera models, described in Section 3.1. For the most frequently used perspective model, we can write

$$r'_l = f \tan \alpha, \tag{7}$$

$$\alpha = \arctan \frac{r'_l}{f}. \tag{8}$$

By combining Equation (6) and Equation (8) we get:

$$dr' = \cos \left[\arctan \frac{r'_l}{f} \right] dr'_l. \tag{9}$$

Total radial distance r' from the point X' on the image plane to the image center F' can be obtained by integration of the Equation (9),

$$\int_0^{r'} dr' = \int_0^{r'_l} \cos \left[\arctan \frac{r'_l}{f} \right] dr'_l, \tag{10}$$

which yields[3]

$$r' = f \cdot ln \left(\frac{r_l}{f} + \sqrt{1 + \frac{r_l^2}{f^2}} \right). \tag{11}$$

This is essentially the distortion function $r' = d(r'_l, \mathbf{p})$, as defined in Equation (5), derived for perspective camera model with accordance to the tilted camera assumption. It is obvious that its parameter vector $\mathbf{p}$ contains only one parameter - focal length f. By solving the Equation (11) for r_l, we can obtain the inverse formula, which defines the

[2]Then, our tilted camera has zero viewing angle, and is distortion free.

[3]Symbolic integration and function inversion were performed using Matlab 5 and its Symbolic Math Toolbox.

transformation of distorted radial distance r' to the undistorted radial distance r'_l in the image plane,

$$r'_l = -\frac{f}{2}\frac{(e^{-\frac{2r'}{f}}) - 1}{e^{-\frac{r'}{f}}}. \tag{12}$$

4 Experiments

We tested the performance of the derived distortion model by using two lenses with focal lengths of 6.5 and 8.5 mm. Several images of calibration pattern were acquired with each of the lens and standard, linear 3D calibration was performed for each lens. Additionally, image of the checkerboard pattern was taken through each of the lens, resulting in grayscale image of 768×576 pixels. Positions of square corners in image pixel coordinates were obtained by convolving the image with the checkerboard operator; several (not more than six) missed points were added manually. Obtained points were grouped into the array of vertical and horizontal lines, which are shown in Figure 4. Four lines (two verical and two horizontal) were chosen for radial distortion evaluation. Two lines pass near the image center and serve as reference, since they are not heavily distorted. Two lines are located at image border and measure the actual improvement in grid linearity. For each of the four lines, marked with asterisks, residual error before and after RMS line fit was measured. Array of pixels was compensated for radial distortion using the formula (12) and measurements were repeated. Tables and graph in Figure 4 show the results.

3D calibration provided us with two focal lengths for each lens (for vertical and horizontal direction) since pixels are not square. The average of those two values in pixels was used for distortion correction as parameter f. Center of distortion was set to the center of image.

Results clearly show that derived distortion model closely resembles radial distortion of both tested lenses. Diagrams in Figure 4e and Figure 4f confirm that radial distortion for border lines decreased significantly. On the other hand, only marginal increase in distortion of center lines can be observed.

5 Conclusion

Derived distortion functions (11) and (12) have built-in implication that the lens radial projection function is close to the perspective projection. Projection function of particular lens can be closer to some other model [5], however, similar derivation could be done for any of the projection functions, provided that the corresponding integral (10) exists.

The distortion functions (11) and (12) need focal length f to model the radial distortion of particular lens. However, unlike the distortion parameters $k_1, k_2 \ldots k_n$, the focal length is closely related to the camera geometry and is as such part of the parameter set of every 3D calibration. Therefore, our distortion correction function introduces *no*

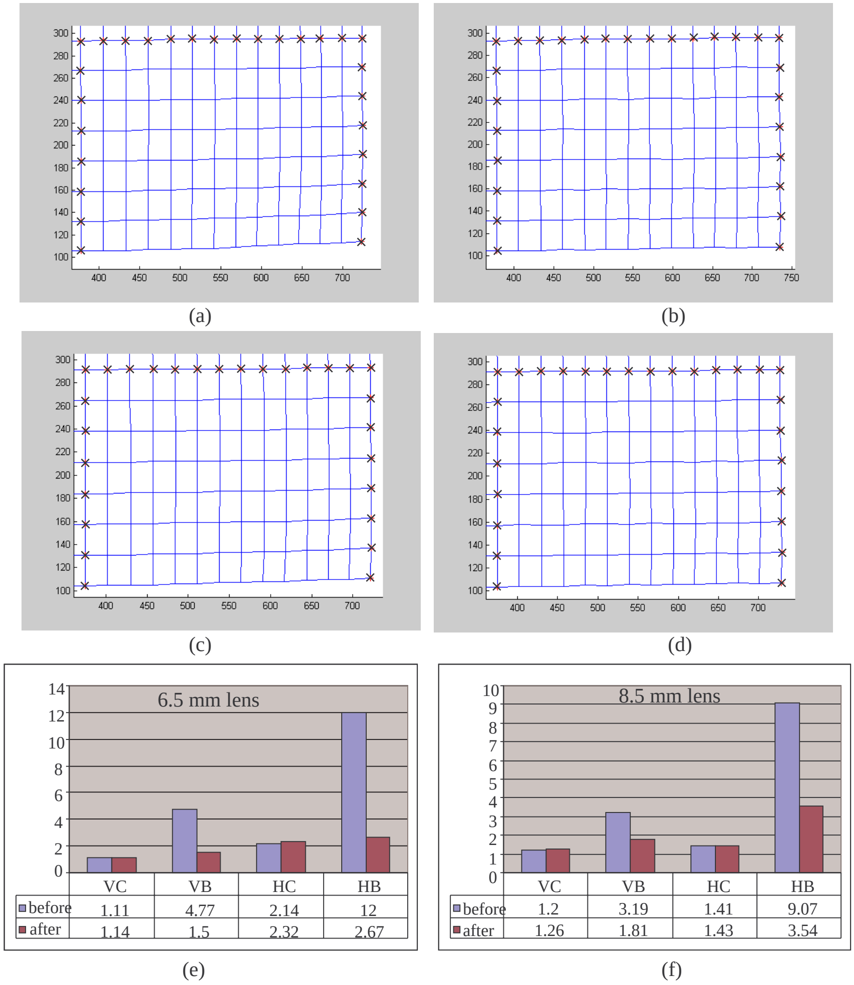


Figure 4: Experiment results. Only lower-right quadrant of the grid is shown. First row: 6.5 mm lens, $f_{average}$=818 pixels. (a) before, (b) after the correction. Second row: 8.5 mm lens, $f_{average}$=1067 pixels. (c) before, (d) after the correction. Third row: numerical results for (e) 6.5 mm lens and (f) 8.5 mm lens. VC - vertical centerline, VB - vertical borderline, HC - horizontal centerline, HB - horizontal borderline. Parts of VC, VB and HC are visible in a) through d) and marked with crosses.

distortion-specific parameters to the camera model. This has important implications. The calibration of lens which have moderate radial distortion can be simplified by not including the radial distortion into the original model. The camera parameters can be then obtained using a closed-form algorithm, for example DLT, and radial distortion can be removed afterwards, with a help of focal length, calculated during the first calibration phase. For wide-angle lenses, the linear camera model can be extended to incorporate radial distortion, which would require iterative nonlinear parameter search, however the dimension of the search space is reduced for at least two parameters of the radial distortion polynomial. Many advanced cameras (for example digital photographic cameras) can measure focal length used for each exposition, and therefore this measurement can be used to reduce radial distortion. In the case that radial distortion correction is desired from purely cosmetical reasons, approximate focal length in pixels can be calculated from the nominal focal length of the lens used and from the dimensions of the image sensor. We successfully employed this technique for some wide-angle images from our lab, however, due to lack of space, results are not presented here.

From the viewpoint of computer vision field, lenses have two important properties: radial projection function and focal length. Both of these properties were taken into the account in the derivation of radial distortion functions, which emphasizes the view that the radial distortion really *is* an inherent property of any lens, *not* an error in manufacturing process or lens design. It is most likely that for some applications the derived radial distortion functions do not provide sufficient accuracy; in this case the need for additional polynomial model remains. However, such polynomial model would probably model the true *errors* of the lens, not the camera and lens *geometry*.

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